Math120R: Precalculus Final Exam Review, Fall 2012

Note: This study aid is intended to help you review for the final exam. It covers the primary concepts in the course. Do not expect this review to be identical to the actual final exam. You should also review tests, notes, and homework problems given throughout the semester.

Multiple Choice Practice

1. Which of the following equations determine y as a function of x?

   (1) $3x + 2y^3 = 10$ (2) $\sqrt{x+1} + y = 8$ (3) $2x - y^2 - 7 = 0$

   (4) $3x^2 - xy = 1$

   (A) all of them (B) 1 and 3 only (C) 1, 2, and 4 only

   (D) 1 and 2 only (E) 1 and 4 only

2. If $f(t) = 3t^2 - 2$, find $f(k-2)$.

   (A) $3k^2 - 12k + 12$ (B) $3k^2 - 8$ (C) $3k^2 - 4k + 2$

   (D) $3k^2 - 4$ (E) None of these

3. If $f(x) = \begin{cases} -x + 4 & \text{if } x < 1 \\ x^2 & \text{if } 1 \le x \le 3 \\ \sqrt{x+3} & \text{if } x > 3 \end{cases}$, what is $f(-2)$?

   (A) 6 (B) 4 (C) 2 (D) 1 (E) None of these

4. Express the area of a rectangle as a function of its width if the width is 25% of its length. Let L and W represent length and width respectively.

(A) $A = (.25W)(W)$ (B) $A = (.75W)(W)$ (C) $A = (4W)(W)$

(D) $A = 4LW$ (E) None of these

5. The equation of the line passing through the ordered pair $(a,0)$ and parallel to the line $x + 2y = 7$ is:

(A) $y = -\frac{1}{2}x + a$ (B) $y = -\frac{1}{2}x + \frac{a}{2}$ (C) $y = \frac{1}{2}x + a$

(D) $y = \frac{1}{2}x + \frac{a}{2}$ (E) None of these

6. Which ONE of the following is an equation of a circle with center $(2,-4)$ passing through the
the
point $(0,-8)$?

(A) $(x - 2)^2 + (y + 4)^2 = \sqrt{20}$

(B) $(x - 2)^2 + (y + 4)^2 = 20$

(C) $(x + 2)^2 + (y - 4)^2 = \sqrt{148}$

(D) $(x + 2)^2 + (y - 4)^2 = 148$

(E) None of these

7. Find the value of c so that the lines $5x + cy = 4$ and $x - 3y = 9$ are perpendicular.

(A) $c = -3$ (B) $c = 15$ (C) $c = -15$

(D) $c = -\frac{5}{3}$ (E) $c = \frac{5}{3}$

8. Which ONE of the following is the solution to the inequality $|-9w + 8| < 6$?

(A) $\left(-\frac{14}{9}, -\frac{2}{9}\right)$ (B) $\left(-\infty, -\frac{2}{9}\right) \cup \left(\frac{2}{9}, \infty\right)$ (C) $\left(\frac{2}{9}, \frac{14}{9}\right)$

(D) $\left(-\infty, \frac{14}{9}\right)$ (E) $\left(-\infty, \frac{2}{9}\right) \cup \left(\frac{14}{9}, \infty\right)$

9. Which of the following is/are true about the circle with equation $(x + 8)^2 + (y - 1)^2 = 7$?

(1) The radius is 49.

(2) $(-8, 1)$ is the center of the circle.

(3) There are no y-intercepts.

(A) 1 and 2 only (B) 2 only (C) 1 and 3 only

(D) 2 and 3 only (E) 3 only

10. Find the equation of the line with $x-$ intercept $(4,0)$ that is perpendicular to the line $3x - 8y = 5$.

(A) $y = -\frac{3}{8}x + \frac{3}{2}$ (B) $y = -\frac{8}{3}x + \frac{32}{3}$ (C) $y = -\frac{8}{3}x + 4$

(D) $y = \frac{3}{8}x + 4$ (E) $y = \frac{8}{3}x - \frac{32}{3}$

11. Solve the equation $x(x+3) = 5$. The EXACT solutions are:

(A) $x = \frac{-3+\sqrt{29}}{2}$ or $x = \frac{-3-\sqrt{29}}{2}$

(B) $x = 5$ or $x = 2$

(C) $x = \sqrt{2}$ or $x = -\sqrt{2}$

(D) $x = 1$ or $x = -4$

(E) $x = \frac{-3+\sqrt{11}}{2}$ or $x = \frac{-3-\sqrt{11}}{2}$

12. Which ONE of the following is the solution to the inequality $y^2(5y+3)(y-6) > 0$?

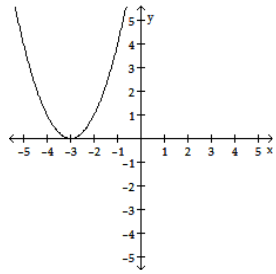
(A) $\left(-\frac{3}{5}, 6\right)$ (B) $\left[-\frac{3}{5}, 6\right]$ (C) $\left(-\infty, -\frac{3}{5}\right) \cup (6, \infty)$

(D) $\left(-\infty, -\frac{3}{5}\right] \cup [6, \infty)$ (E) all real numbers

13. Which of the following represent(s) y as a function of x?

(1) $3y^2 = 5x$

(2)



(3)

x	y
1	5
2	4
3	5
4	4

(A) 1 only (B) 1 and 3 only (C) 2 only

(D) 2 and 3 only (E) All of them

14. Find the domain of the function $f(t) = \sqrt{81 - t^2}$.

(A) $(-\infty, -9) \cup (9, \infty)$

(B) $(-\infty, -9] \cup [9, \infty)$

(C) $(-9, 9)$

(D) $[-9, 9]$

(E) all real numbers

15. An equation of a graph obtained from vertically shrinking the graph of $y = \sqrt{x}$ then shifting the graph up twenty units is:

(A) $y = \frac{5}{3}\sqrt{x} + 20$ (B) $y = \sqrt{\frac{7}{2}x + 20}$ (C) $y = \frac{3}{4}\sqrt{x} + 20$

(D) $y = 2\sqrt{x + 20}$ (E) None of these

16. If $f(x)$ is a function with domain $[-8, 12]$, find the domain of $\frac{1}{2}f(x-3)$.

(A) $[-5,6]$ (B) $[-7,12]$ (C) $[-5,15]$

(D) $[-11,9]$ (E) $[-13,12]$

17. You can get the graph of $y = -g(2x)$ by transforming the graph of $y = g(x)$ in the following way:

    (A) Shrink horizontally and reflect across the x -axis.

    (B) Shrink horizontally and reflect across the y -axis.

    (C) Stretch vertically and reflect across the x -axis.

    (D) Stretch vertically and reflect across the y -axis.

    (E) None of these

18. If $(-4,7)$ is a point on the graph of $y = h(t)$, which of the following must be a point on the graph of $y = h(-t) - 2$?

    (A) $(-4,-9)$ (B) $(-4,-5)$ (C) $(4,5)$ (D) $(4,9)$ (E) None of these

19. Find the vertex of the quadratic function $f(x) = \frac{4}{7}x^2 - \frac{16}{7}x + 3$. The y - coordinate of the vertex is:

    (A) $\frac{1}{2}$ (B) $\frac{5}{7}$ (C) $\frac{6}{7}$ (D) 1 (E) None of these

20. A horticulturist has determined that the number of inches a young oak tree grows in one year is a function of the annual rainfall r given by $g(r) = -0.01r^2 + 0.1r + 2$. What is the maximum number of inches a young oak can grow in one year? The maximum number of inches is:

(A) less than 1 (B) between 1 and 2 (C) between 2 and 3

(D) between 3 and 4 (E) between 4 and 5

21. The sum of the base and the height of a triangle is 20 centimeters. Find the maximum area of the triangle.

(A) 20 square centimeters

(B) 50 square centimeters

(C) 100 square centimeters

(D) 200 square centimeters

(E) 400 square centimeters

22. Which of the following about $g(r) = 4r^2 + 15r + 14$ is/are true?

(1) The vertex of g is below the $r-$ axis.

(2) g is an even function.

(3) $r = -\frac{15}{8}$ is the axis of symmetry.

(A) 3 only (B) 2 only (C) 2 and 3 only

(D) 1 and 3 only (E) All of them

23. A rectangle with length L and width W has a diagonal of 10 inches. Express the perimeter P of the rectangle as a function of L.

(A) $P = 10L - 2L^2$

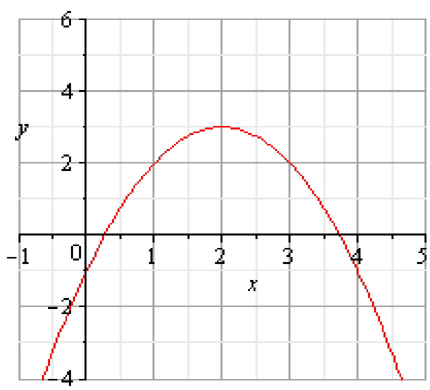
(B) $P = 2L + 2\sqrt{100 - L^2}$

(C) $P = L(\sqrt{100 - L^2})$

(D) $P = 2L - 100$

(E) $P = L^2 + W^2 + 100$

24. Find the equation of the parabola whose graph is shown below. The coefficient of x^2 is a number:



vertex = (2,3)

(A) between -3 and -1.5

(B) between -1.5 and 0

(C) between 0 and 1.5

(D) between 1.5 and 3

(E) None of these

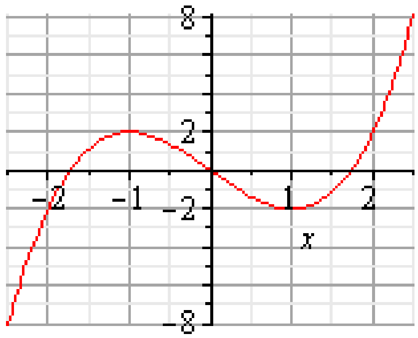
25. Which of the following functions is/are neither even nor odd?

(1) $f(x) = |23x|$

(2) $g(p) = \dfrac{5p}{p^2 + 2}$

(3) $h(t) = 5 - 4t - 2t^3$

(4) $y = f(x)$



(A) All of them (B) 2 and 3 only (C) 2 only

(D) 3 only (E) None of them

26. Given $f(x) = 6x + 3$ and $g(x) = \sqrt{x}$, find $(f \circ g)(x)$.

(A) $\sqrt{6x+3}$ (B) $6\sqrt{x} + 3$ (C) $\sqrt{3} + \sqrt{6x}$

(D) $6x\sqrt{x} + 3\sqrt{x}$ (E) None of these

27. Given the functions $f(w) = \sqrt{5-2w}$ and $g(w) = w^2 - 4$, find $h(w) = (f \circ g)(w)$.

(A) $h(w) = -2w + 9$ (B) $h(w) = \sqrt{-2w^2 + 1}$

(C) $h(w) = (\sqrt{5-2w}) \cdot (w^2 - 4)$ (D) $h(w) = -2w + 1$

(E) $h(w) = \sqrt{-2w^2 + 13}$

28. Find $Q^{-1}(t)$ if $Q(t) = \dfrac{C}{4t-1}$. (C is a nonzero real number)

(A) $Q^{-1}(t) = \dfrac{C}{4}t + C$ (B) $Q^{-1}(t) = \dfrac{4t-1}{C}$ (C) $Q^{-1}(t) = \dfrac{C+t}{4t}, t \neq 0$

(D) $Q^{-1}(t) = \dfrac{C-4t}{t}, t \neq 0$ (E) None of these

29. Suppose $g(4) = 30$ means the volume of water in a container is 30 ounces when the depth of the water is 4 inches. What is the meaning of $g^{-1}(50) = 10$?

(A) The volume of the water is 10 ounces when the depth of the water is 50 inches.

(B) The depth of the water is 10 inches when the volume of the water is 50 ounces.

(C) The depth of the water is 0.2 inches when the volume of the water is 50 ounces.

(D) The volume of the water is 5 ounces when the depth of the water is 10 inches.

(E) None of these

30. If $f(x)$ is a one-to-one function, and $f(8) = 11$, then which of the following CANNOT be true?

(A) $f(11) = 8$ (B) $f^{-1}(11) = 8$ (C) $f^{-1}(5) = 3$

(D) $f^{-1}(11) = 5$ (E) $f(-8) = -11$

31. Let $f(x) = a(x-b)^3(x-c)^4$, where b and c are real numbers and $a < 0$. Which ONE of
the following represents the correct end behavior of $f(x)$?

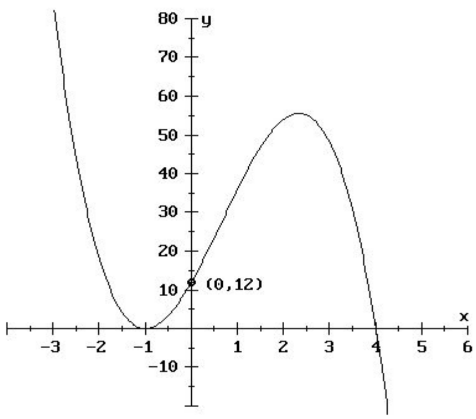
(A) $y \to \infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

(B) $y \to -\infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

(C) $y \to \infty$ as $x \to \infty$
$y \to -\infty$ as $x \to -\infty$

(D) $y \to -\infty$ as $x \to \infty$
$y \to \infty$ as $x \to -\infty$

32. The graph of $y = h(x)$ is given below. Which ONE of the following is the correct equation
for $h(x)$?



(A) $h(x) = 3(x+1)(x-4)$ (B) $h(x) = -\frac{1}{3}(x+1)^2(x-4)$

(C) $h(x) = -3(x+1)^2(x-4)$ (D) $h(x) = -\frac{1}{3}(x+1)(x-4)$

(E) $h(x) = 3(x-1)^2(x+4)$

33. Find all the real zeros of $f(x) = x^3 + 5x^2 + 7x + 2$. The largest real zero is:

(A) $\frac{-3+\sqrt{5}}{2}$ (B) -0.5 (C) $\frac{-3+\sqrt{13}}{2}$

(D) $\frac{-3+\sqrt{7}}{2}$ (E) -2

34. Which of the following MUST be true?

(1) A polynomial of degree 4 has 4 unique zeros.

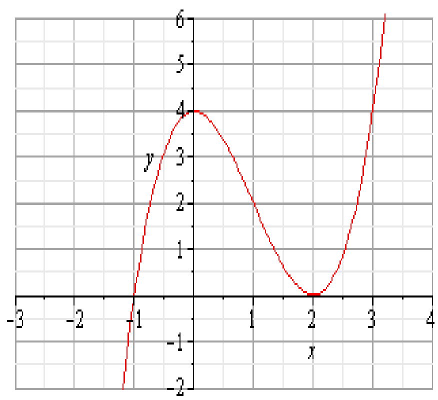
(2) A polynomial of degree 5 has at least 1 real zero.

(3) A polynomial of degree 2 has at least 1 real zero.

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 2 only (E) 1 and 3 only

35. Which of the following could be the equation of the polynomial $P(x)$ graphed below?



(A) $P(x) = (x+2)^2(x-1)$ (B) $P(x) = 2(x-2)^2(x+1)$

(C) $P(x) = (x-2)^2(x+1)$ (D) $P(x) = -2(x-2)^2(x+1)$

(E) None of these

36. Which of the following statements is/are equivalent to "$x+5$ is a factor of the polynomial $f(x)$"?

(1) $x = 5$ is a solution to $f(x) = 0$.

(2) $x = -5$ is a zero of $f(x)$.

(3) $(-5,0)$ is an x-intercept of $f(x)$.

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 3 only (E) 2 and 3 only

37. What is the remainder when $p(x) = x^4 + x^3 - x^2 - 2$ is divided by $x - 3$?

(A) -26 (B) -17 (C) 0 (D) 43 (E) None of these

38. Let $f(r) = \dfrac{r^2 + 5}{12r^2 - 7}$. Which ONE of the following statements is true?

(A) $f(r)$ has exactly two vertical asymptotes.

(B) $y = 0$ is a horizontal asymptote.

(C) The domain of $f(r)$ is all real numbers.

(D) $f(r)$ has a slant asymptote.

(E) $f(r)$ has exactly one vertical asymptote.

39. Find the vertical asymptote(s), if any, for $f(x) = \dfrac{x - 2}{3x^2 - 5x}$.

(A) $x = 2$ (B) $x = 0$ (C) $x = 0$ and $x = \dfrac{5}{3}$

(D) $x = \dfrac{5}{3}$ (E) There are no vertical asymptotes.

40. Which of the following statements is/are true about $g(t) = \dfrac{-t^2 + 6}{4t^2 - 7t + 2}$?

(1) $g(t) \to -\frac{1}{4}$ as $t \to \infty$

(2) $g(t)$ has a slant asymptote.

(3) 0 is in the range of $g(t)$

(A) 1 only (B) 2 only (C) 3 only

(D) 1 and 2 only (E) 1 and 3 only

41. Find the slant asymptote of $f(x) = \frac{2x^2 + 7}{x + 3}$.

(A) $y = 2x - 6$ (B) $y = 2x + 1$ (C) $y = 2$

(D) $y = 2x - 10$ (E) None of these

42. Let $f(x) = C \cdot b^x$. Determine constants C and b so that $f(-1) = 10$ and $f(2) = \frac{5}{4}$.

(A) $C = 5, b = \frac{1}{2}$ (B) $C = 10, b = \frac{1}{4}$ (C) $C = 2, b = 5$

(D) $C = 1, b = \frac{5}{4}$ (E) None of these

43. Which ONE of the following is true about the graph of $y = 5000e^{-0.0002x} - 9000$?

(A) $y \to \infty$ as $x \to -\infty$ and $y \to 9000$ as $x \to \infty$.

(B) $y \to \infty$ as $x \to -\infty$ and $y \to -9000$ as $x \to \infty$.

(C) $y \to 5000$ as $x \to -\infty$ and $y \to -9000$ as $x \to \infty$.

(D) $y \to 0$ as $x \to -\infty$ and $y \to \infty$ as $x \to \infty$.

(E) $y \to 5000$ as $x \to -\infty$ and $y \to 0$ as $x \to \infty$.

44. The domain of $y = \log_7(4 - 3x)$ is:

(A) $(\frac{4}{3},\infty)$ (B) $(0,\infty)$ (C) $(-\infty,\frac{4}{3})$

(D) $[\frac{4}{3},\infty)$ (E) $(-\infty,\infty)$

45. Find the domain of $g(r) = \ln(r^2 - 1)$.

(A) $(-1,1)$

(B) $[-1,1]$

(C) $(-\infty,-1)\cup(1,\infty)$

(D) $(-\infty,-1]\cup[1,\infty)$

(E) All real numbers

46. Which of the following statements is/are true about $f(x) = \log_b(x)$? $(b > 0, b \neq 1)$

(1) The domain is $(0,\infty)$

(2) $(0,1)$ is the y-intercept

(3) The range is $(-\infty,\infty)$

(4) The x-intercept is $(b,0)$

(A) 2 and 3 only (B) 1 and 3 only (C) 2, 3, and 4 only

(D) 1, 2, and 4 only (E) 1 only

47. Find the x-intercept of the graph of $y = \ln(x - k) + 2$.

(A) $(e^{-2}+k,0)$ (B) $(\ln(-k)+2,0)$ (C) $(e+k,0)$

(D) $(e^{k-2},0)$ (E) None of these

48. Express as a single logarithm and simplify if possible:

$$\frac{1}{3}\log x + 4\log y - 2\log z$$

(A) $\log(\frac{1}{3}x + 4y - 2z)$ (B) $\log(x^{\frac{1}{3}} + y^4 + z^3)$ (C) $\frac{7}{3}\log(\frac{xy}{z})$

(D) $\log(\frac{x^{\frac{1}{3}}y^4}{z^2})$ (E) None of these

49. Let $H(t) = 5 + 2e^{-t}$. Find $H^{-1}(9)$.

(A) $5+2e^{-9}$ (B) $\ln(7)$ (C) $-\ln(2)$

(D) $-\ln(7)$ (E) $\frac{1}{5+2e^{-9}}$

50. Let $f(t) = \log_3(9t) + 4$ and $g(t) = 3^t$. Simplify $(f \circ g)(t)$ completely.

(A) $3t+4$ (B) $27^t + 4$ (C) $3t+6$

(D) $t+4$ (E) $t+6$

51. Using your calculator, solve the equation $\log x = 4 - x$. The solution is a number:

(A) Less than 3.2

(B) Between 3.2 and 3.3

(C) Between 3.3 and 3.4

(D) Between 3.4 and 3.5

(E) Greater than 3.5

52. Solve for x: $3^x = 5^{x-1}$. The solution is a number:

(A) between 2 and 4 (B) between -5 and -3

(C) between -1 and 0 (D) between -3 and -1

(E) None of these

53. A present value of $2600 is invested in an account with an annual interest rate of 4.1%. Determine the minimum amount of time required for the present value to triple, assuming the interest is compounded monthly. The minimum amount of time required is:

(A) less than 26 years

(B) between 26 and 27 years

(C) between 27 and 28 years

(D) between 28 and 29 years

(E) between 29 and 30 years

54. How much MORE money will you earn in an account that compounds interest continuously than in an account compounds interest quarterly if you invest \$3000 for 7 years at an interest rate of 11%?

(A) \$67.02 (B) \$59.37 (C) \$101.16 (D) \$32.52 (E) None of these

55. In 1980, the population of the United States was approximately 226.5 million people. In 1990, the population had grown to approximately 246.7 million people. Assuming an exponential growth model $n(t) = n_0 e^{rt}$, what is the projected population of the U. S. in the year 2000?

(A) Less than 260 million people.

(B) Between 260 and 265 million people.

(C) Between 265 and 270 million people.

(D) Between 270 and 275 million people.

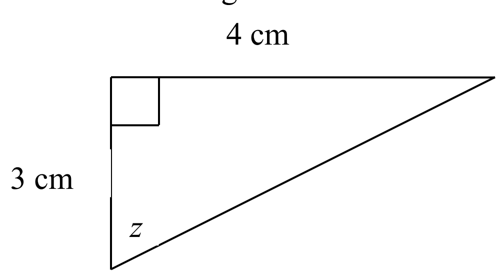
(E) More than 275 million people.

56. The release of fluorocarbons used in household sprays destroys the ozone layer in the upper atmosphere. Suppose the amount of ozone is given by $m = m_0 e^{-0.0025t}$, where t is measured in years. How long will it take for 70% of the ozone to disappear, rounded to the nearest year?

(A) 143 years (B) 1699 years (C) 1360 years

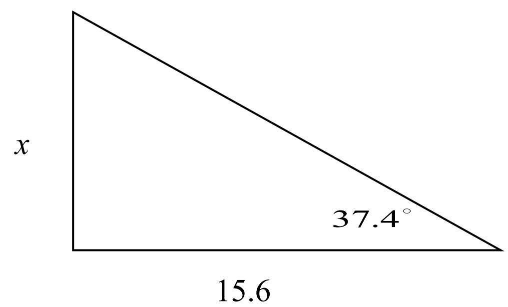
(D) 482 years (E) None of these

57. Find z as a degree measure rounded to one decimal place.



(A) 41.4° (B) 48.6° (C) 36.9° (D) 53.1° (E) None of these

58. Use the angle 37.4° to determine the exact value of x in the figure below.



(A) $x = 15.6\sin(37.4^\circ)$

(B) $x = \dfrac{\tan(37.4^\circ)}{15.6}$

(C) $x = 15.6\cot(37.4^\circ)$

(D) $x = \dfrac{\cot(37.4^\circ)}{15.6}$

(E) $x = 15.6\tan(37.4^\circ)$

59. Find the period of $y = -4\cot(2x)$

(A) π (B) $\dfrac{\pi}{2}$ (C) 2π (D) $\dfrac{\pi}{4}$ (E) None of these

60. Simplify the expression $\dfrac{1}{\cos^2\theta} - 1$ completely. The result is

(A) $\cot^2\theta$ (B) $\sec^2\theta$ (C) 0 (D) $\tan^2\theta$ (E) None of these

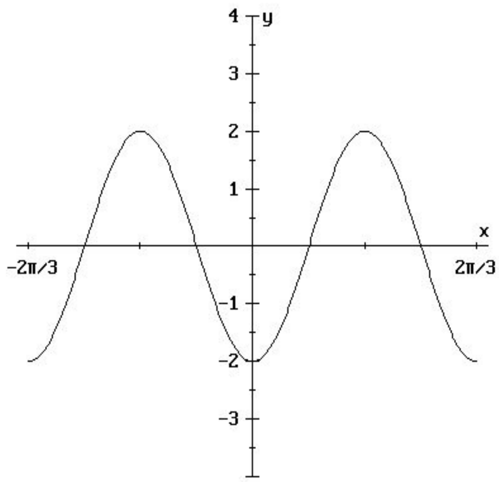
61. Simplify $\dfrac{\sec x}{\csc x}$ completely.

(A) 1 (B) $\tan x$ (C) $\tan^3 x$ (D) $\cot x$ (E) $\cot^2 x$

62. If the radius of a circle is 3 cm., then the measure of an angle that cuts an arc of length 6 cm is:

(A) 120° (B) π (C) 2 (D) 30º (E) 0.5

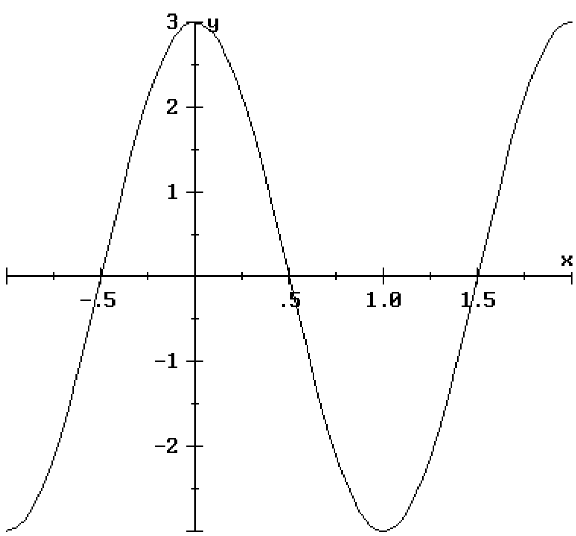
63. Which of the following could be the equation of the function $g(x)$ graphed below?



(A) $g(x) = -2\cos(3x)$ (B) $g(x) = -2\sin(3x)$ (C) $g(x) = -2\cos\left(\frac{x}{3}\right)$

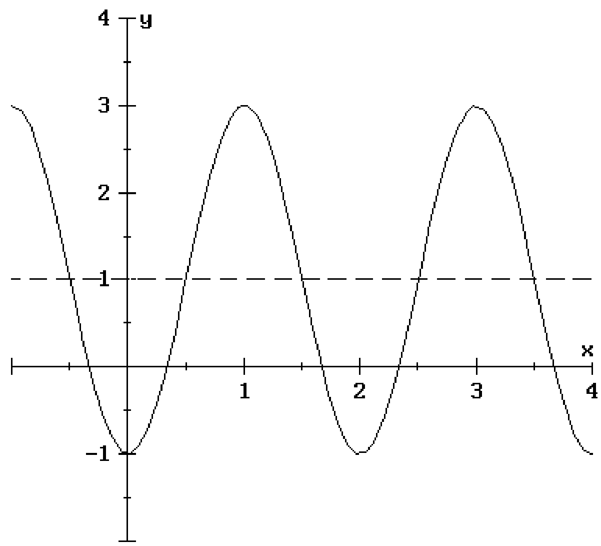
(D) $g(x) = -2\sin\left(\frac{x}{3}\right)$ (E) $g(x) = 2\sin(3x)$

64. The graph below shows $y = A\cos(Bx)$. What is the value of B ?



(A) 4π (B) 3π (C) 2π (D) π (E) 2

65. The equation of the graph below is

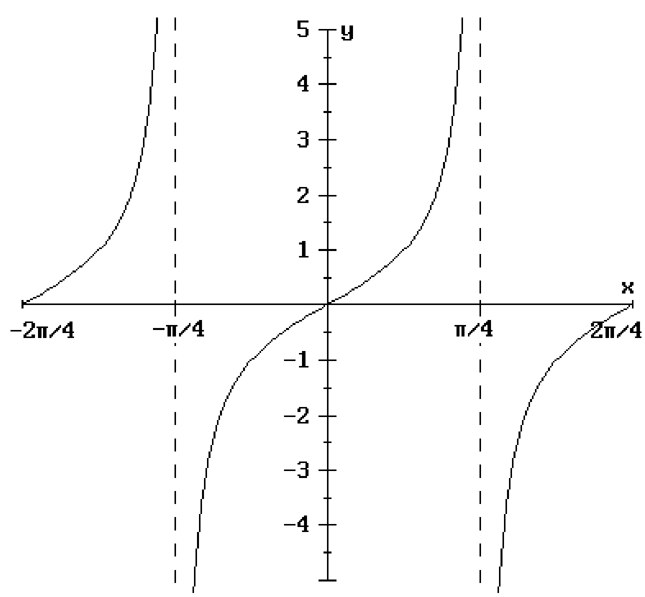


(A) $f(x) = 2 + 3\cos(2x + 2)$ (B) $f(x) = 1 + 2\cos(\pi x + \pi)$

(C) $f(x) = 1 + 2\sin(\pi x - \pi)$ (D) $f(x) = 2 + 3\sin(2x - 2)$

(E) None of these

66. Find the equation of the following graph.



(A) $y = \tan \frac{\pi}{2} x$ (B) $y = \tan 4x$ (C) $y = \tan 2x$

(D) $y = \tan \frac{1}{2} x$ (E) None of these

67. A loudspeaker diaphragm is oscillating in simple harmonic motion described by the equation $y = a\cos(\omega t)$ with a frequency of 520 cycles per second. Find the value of ω .

(A) 260

(B) 260π

(C) 1040

(D) 1040π

(E) None of these

68. Suppose the angle A terminates in Quadrant III and $\sin A = -\dfrac{3}{5}$. Find $\cos A$.

(A) $-\dfrac{4}{5}$ (B) $\dfrac{4}{5}$ (C) $\dfrac{3}{5}$ (D) $-\dfrac{3}{5}$ (E) None of these

69. Let θ be an angle in standard position. The terminal side of θ intersects the unit circle at $\left(-\dfrac{2}{5}, \dfrac{\sqrt{21}}{5}\right)$. Find $\cot\theta$.

(A) $\sqrt{21}$ (B) $\dfrac{\sqrt{21}}{5}$ (C) $-\dfrac{1}{5}$ (D) $-\dfrac{2}{\sqrt{21}}$ (E) $-\sqrt{21}$

70. Suppose the angle A terminates in Quadrant II and $\sin A = \dfrac{3}{5}$. Find $\tan A$.

(A) $\dfrac{3}{4}$ (B) $-\dfrac{3}{4}$ (C) $\dfrac{4}{3}$ (D) $-\dfrac{4}{3}$ (E) None of these

71. Find $\cos\left(\dfrac{7\pi}{6}\right)$ exactly.

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{-\sqrt{3}}{2}$ (C) $-\frac{1}{2}$ (D) $\frac{1}{2}$ (E) None of these

72. Find $\sec\left(\frac{-4\pi}{3}\right)$ exactly.

(A) 2 (B) $\frac{2}{\sqrt{3}}$ (C) $-\frac{2}{\sqrt{3}}$ (D) $-\frac{1}{2}$ (E) None of these

73. Suppose $x = n$ with $0 \le n \le \frac{\pi}{2}$ is a solution to the equation $\sin x = m$. Find another solution to the equation $\sin x = m$.

(A) $-n$ (B) $2\pi - n$ (C) $\pi - n$ (D) $\pi + n$ (E) None of these

For problems 74 and 75, consider the function $f(t) = -5\sin 2(t+4) + 6$.

74. The amplitude is:

(A) –5 (B) –4 (C) 2 (D) 5 (E) None of these

75. The maximum value is:

(A) **1** (B) 4 (C) 6 (D) 11 (E) None of these

76. If cos $m = n$ and sin $m = k$, then

(A) $\cos(-m) = -n$ and $\sin(-m) = -k$ (B) $\cos(-m) = n$ and $\sin(-m) = k$

(C) $\cos(-m) = -n$ and $\sin(-m) = k$ (D) $\cos(-m) = n$ and $\sin(-m) = -k$

(E) None of these

77. On what interval is the identity $\sin^{-1}(\sin x) = x$ valid?

(A) $[0, 2\pi]$ (B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (C) $[0, \pi]$ (D) **[−1,1]** (E) None of these

78. Let $\sin x = \frac{4}{5}$ where x is in Quadrant II. Find $\sin(2x)$.

(A) $-\frac{24}{25}$ (B) $-\frac{12}{25}$ (C) $\frac{6}{5}$

(D) $\frac{12}{25}$ (E) $\frac{24}{25}$

79. Suppose $\cos(x) = \frac{1}{2}$ and $\sin(y) = \frac{1}{2}$ where x terminates in Quadrant I and y terminates in Quadrant II. Find $\sin(x - y)$.

(A) -1 (B) $-\frac{1}{4}$ (C) 0

(D) $\frac{1}{2}$ (E) 1

80. What is the domain of $y = \tan^{-1} x$?

(A) $(-\infty,\infty)$ (B) $[0, 2\pi]$ (C) $[0, \pi]$ (D) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (E) None of these

81. Simplify the expression $\tan\left(\sin^{-1}(x)\right)$.

(A) $\sqrt{1+x^2}$ (B) $\frac{x}{\sqrt{1-x^2}}$ (C) x (D) $\sqrt{1-x^2}$ (E) $\frac{\sqrt{1+x^2}}{x}$

82. How many distinct solutions does the equation $\sin(3x) = \frac{1}{2}$ have on the interval $[0, 2\pi)$?

(A) 1 (B) 3 (C) 6 (D) 12 (E) Infinitely many

Short Response Practice

1. The relationship between the tuition in dollars, T, and the number of credits, c, at a particular college is given by

$$T(c) = \begin{cases} 100 + 120c & 0 \le c \le 6 \\ 800 + 120(c - 6) & 6 < c \le 18 \end{cases}$$

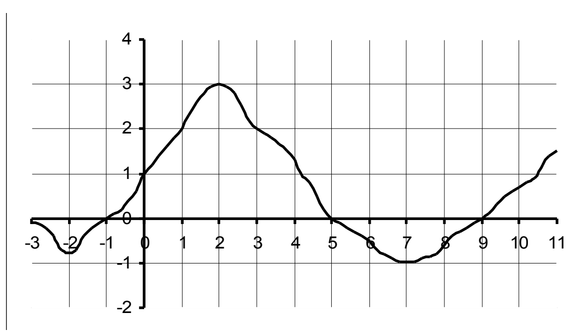
a. What is the tuition for 7 credits?

b. If the tuition was $1880, how many credits were taken?

c. What is the domain of this function?

d. What are the practical interpretations of the y - intercept and the slope?

2. Use the graph at the right to answer the questions below.



a. Find $f(0)$.

b. On what intervals is $f(x)$ increasing?

c. Find x so that $f(x) = 2$.

d. Find the zeros of $f(x)$.

e. What is $f(f(9))$?

3. Let $f(t) = -t^2 + 7t - 5$. Find and simplify $\dfrac{f(t+h) - f(t)}{h}$.

4. Let $f(x) = 5x^2 - 9$. Find and simplify $\dfrac{f(x+h) - f(x-h)}{2h}$.

5. Let $G(v)$ be a linear function representing the grade point average of a high school student who plays video games for v minutes a day. When a student plays video games for 85 minutes a day, the student's grade point average is 2.89. When a student plays video games for 155 minutes a day, the student's grade point average is 2.05.

   a. Determine the formula for the linear function $G(v)$.

   b. Give a practical interpretation of the slope in terms of grade point average and amount of time spent playing video games.

6. Suppose the tax bill $T(y)$ of a single person whose adjusted gross income is y dollars is represented by a linear model. Assume the tax bill for a person with $9550 in adjusted gross income is $1015, and the tax bill for a person with $19650 in adjusted gross income is $2530.

   a. Determine the formula for $T(y)$.

   b. Explain what the average rate of change of T tells you in terms of the tax bill and adjusted gross income.

7. A cereal company determines that there is a linear relationship between A, the amount it spends on advertising, and W, the number of boxes of cereal it sells. If the company spends $31,412 on advertising, the company will sell 84,525 boxes of cereal. If the company spends

$22,025 on advertising, the company will sell 62,175 boxes of cereal.

a. Find a formula for the linear function $A = f(w)$

b. Give a practical interpretation of the slope in terms of the amount spent on advertising and the number of boxes of cereal sold.

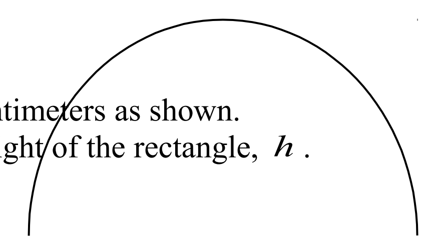
8. A piece of wire 20 cm long is cut into two pieces. The first is bent into a circle, the second into a square. Express the combined total area of the circle and square as a function of x, where x represents the length of the wire that is bent into a circle.

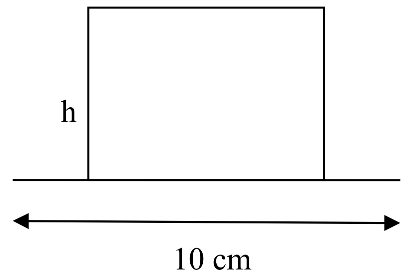
9. The volume of a right circular cone is given by the formula $V = \frac{1}{3}\pi r^2 h$. If the volume of a right circular cone is 50 cubic centimeters, express the radius as a function of height.

10. An airplane manufacturer can produce up to 15 planes per month. The profit made from the sale of these planes can be modeled by $P(x) = -0.2x^2 + 4x - 3$ where $P(x)$ is the profit in hundred thousand of dollars per month and x is the number of planes made and sold. Based on this model, how many planes should be made and sold to maximize the profit? What is the maximum profit?

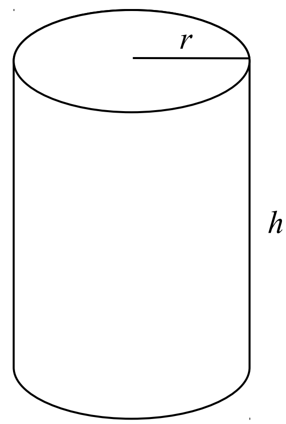
11. An open top rectangular box with a square bottom has a volume of 150 cubic meters. Its bottom and sides are made from two different materials. It costs 8 dollars per square meter for the bottom material, and 10 dollars per square meter for the sides. Express the total cost of building the box in terms of y, where y represents one of the lengths of the bottom side.

12. A rectangle is inscribed in a semicircle with diameter 10 centimeters as shown. Express the area of the rectangle, A, as a function of the height of the rectangle, h. Include units in your answer.





13. A closed cylindrical can has a volume of 22 cubic inches.



Express the surface area, A , of the can as a function of its radius r . Your final answer must be simplified.

14. A farmer introduces 100 trout into his pond. The population of the trout can be modeled by the function $p(t) = \dfrac{150t + 100}{0.04t + 1}$, where t is time measured in months. Find and give an interpretation of the horizontal asymptote of $p(t)$.

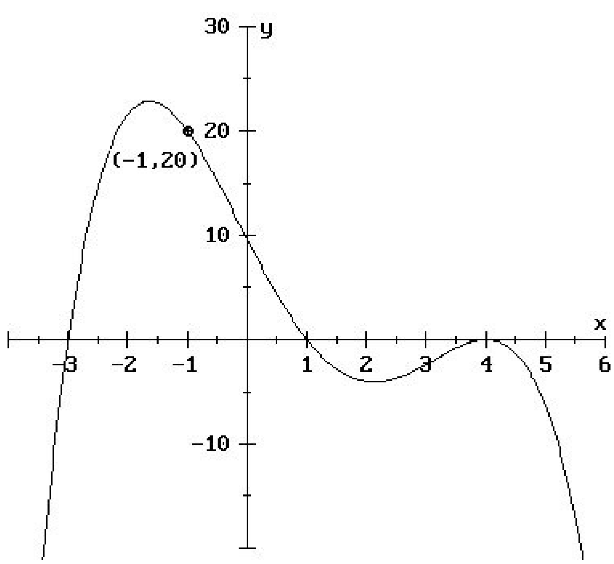
15. A rational function $f(x)$ has one vertical asymptote $x = 2$, and one horizontal asymptote,
$y = -3$. Also, the graph of $f(x)$ passes through the ordered pair $(5,0)$. Determine an equation for $f(x)$ with these properties.

16. A spherical balloon is being inflated. Suppose the radius of the balloon is increasing at a rate of 2 centimeters per second.

a. Express the radius r of the balloon as a function of time t.

b. Express the volume V of the balloon as a function of time t.

17. Determine a possible formula for the polynomial function $y = g(x)$ whose graph is shown below. Leave your answer in factored form.



18. Let $f(x) = \dfrac{-3x + 7}{2x + c}$

a. Find $f^{-1}(x)$.

b. Determine the value of C so that $x = -9$ is a vertical asymptote of $f(x)$.

19. Let $f(x) = \dfrac{3x^2 + 5}{(Bx + 1)(x - 2)}$.

a. Determine the value of B so that $y = \dfrac{1}{4}$ is a horizontal asymptote of $f(x)$.

b. Determine the value of B so that $f(x)$ has a slant asymptote. Find the equation of the slant asymptote.

20. The formula $K = \dfrac{F - 32}{1.8} + 273.15$ is used to determine the corresponding Kelvin measure K of a temperature with Farhenheit measure F. Find and simplify the inverse of K. Give a practical interpretation of the inverse of K.

21. The percent of the United States population that uses the Internet can be modeled by the logistic function given by $P(y) = \dfrac{73.9}{1 + 5.4e^{-0.415y}}$, where y is the number of years after 1995.

a. Using the model above, find the percentage of Internet users in the United States in 2001, rounded to 2 decimal places.

b. In what year would the percentage of Internet users in the United States reach 68.1%? You must show your work algebraically in order to receive full credit.

22. A cup of boiling water with temperature 100 degrees Celsius is placed outside where the air temperature is 25 degrees Celsius. After 13 minutes, the temperature of the water is 74 degrees Celsius. The model $T(t) = T_S + D_0 e^{-kt}$ gives the temperature of an object after time t, where T_S is the temperature of the object's environment and D_0 is the initial temperature difference between the object and its environment.

a. Determine an approximate model that gives the temperature of the water after t minutes. Round any constants in your model to 4 decimal places.

b. Use the model you created in part (a) to determine how long it will take for the temperature of the water to be 40 degrees Celsius. Use algebraic methods to determine your answer. Round your answer to 2 decimal places. Include units in your answer.

23. Suppose the population of a town is 23,000 people in the year 1995. If the population grows at a relative rate of 5.4% per year, find a model giving the population t years after 1995.

24. Determine the shortest distance from the ordered pair $(1,4)$ to the graph of $y = x^2$. Approximate your answer to the nearest hundredth.

25. Suppose $\cos t = \dfrac{2}{3}$ and that the terminal point of t is in Quadrant IV.
Use the fundamental identities to find the exact value of $\sin t, \csc t,$ and $\tan t$.

26. Solve for the indicated variable. Give exact answers.

a. $\log(y-1) + \log(y+1) = \log 8$

b. $e^{(5x-2)} + 9 = 13$

c. $2\sin\theta\cos\theta = \dfrac{\sqrt{3}}{2}$ on the interval $[0,2\pi)$

d. $3\sin^2 t = 2\sin t + 1$ on the interval $[0,2\pi)$

e. $\sin(2\theta) + \cos\theta = 0$ where $0 \le \theta \le 2\pi$

f. $|-4w+3| = 11$

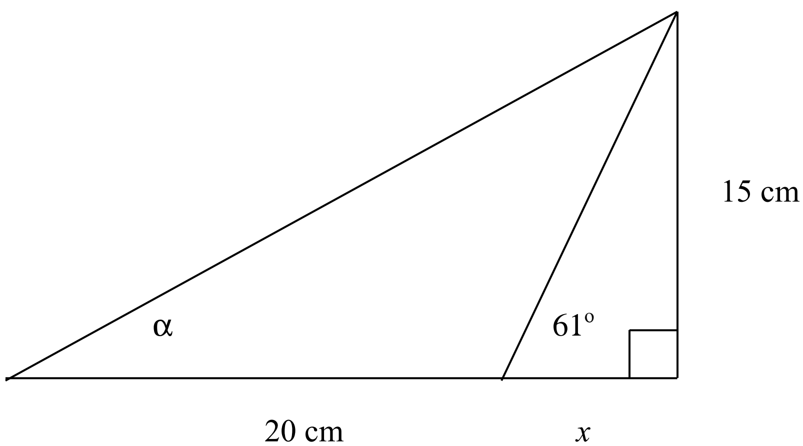
g. $\cos^2\theta = \dfrac{3}{4}$ where $0 \le \theta < 2\pi$

h. $\ln(7w+3) = 11$

i. $2\sin^2 t + \sin t = 1$ where $0 \le t < 2\pi$

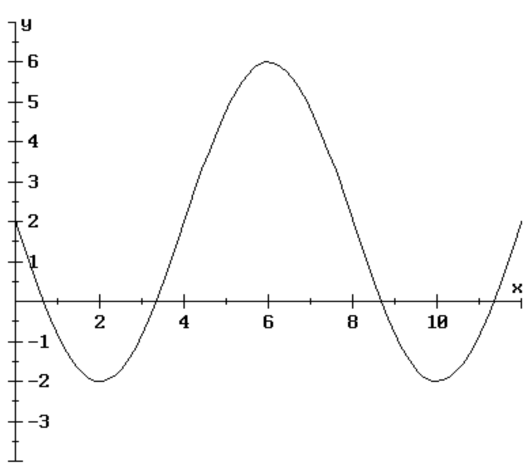
j. $2(\log_5 t)^2 - 3\log_5(t) = 2$

27. Find α and *x*. Approximate your answers to the nearest hundredth.

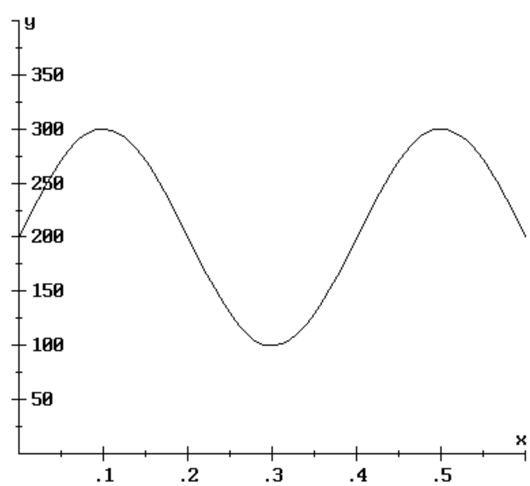


28. Find a possible formula for each graph

a.



b.



29. Simplify each expression. Your final answer should be exact and should not contain any trigonometric expression.

a. $\cos(\tan^{-1}(2))$ b. $\sin(\cos^{-1}(x))$

30. Suppose $\tan\alpha = -\frac{7}{3}$, where α terminates in Quadrant II.

a. Find $\cos\alpha$. Give an **exact** answer.

b. Find $\cos(2\alpha)$. Give an **exact** answer.

31. The top floor of a building undergoes damped harmonic motion after a sudden brief earthquake. At time $t = 0$, the displacement is at a maximum, 16 centimeters from the normal position. The damping constant is $c = 0.72$ and the building vibrates at 1.4 cycles per second.

a. Find a function of the form $y = ke^{-ct}\cos\omega t$ to model the motion.

b. What is the displacement after 10 seconds?

32. Simplify the following trigonometric expressions:

a. $\cos\theta(\tan\theta + \cot\theta)$

b. $\frac{1-\sin t}{\cos t} + \frac{\cos t}{1-\sin t}$

c. $\dfrac{\tan^2 w - \tan w}{1 - \sec^2 w}$